Improving Image Reconstruction Accuracy Using Discrete Orthonormal Moments

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Abstract – *Several pattern recognition applications use orthogonal moments to capture independent shape characteristics of an image, with minimum amount of information redundancy in a feature set. Legendre, Zernike, and Pseudo-Zernike moments are examples of such orthogonal feature descriptors. An image can also be reconstructed from a sufficiently large number of orthogonal moments. Discrete orthogonal moments provide a more accurate description of image features by evaluating the moment components directly in the image coordinate space. This paper examines some of the problems associated with the computation of large order Tchebichef moments, and proposes an orthonormal version to improve the quality of reconstructed images.*

Keywords: Tchebichef moments, discrete orthogonal moments, orthonormal moments, image reconstruction.

1. Introduction

Discrete orthogonal moments are powerful tools for characterizing image shape features for applications in pattern recognition, and identification. They provide significant advantages over moments based on continuous orthogonal functions. For example, the computation of Legendre and Zernike moments require the transformation of image coordinates to a region within the unit square, and the discretization of the continuous integrals. Numerical approximation errors involved in such computations can be significantly large in higher order moments, and may affect the orthogonality property of the moments. Consequently, the quality of image reconstruction is affected even though the basis set is strictly orthogonal in its domain.

Tchebichef (sometimes also written as Chebyshev) moments were introduced as image feature descriptors, in [1]. Tchebichef polynomials are the simplest among discrete orthogonal functions of unit weight [2,3,4], and therefore many of the analytical properties of the corresponding moments can be easily derived, and compared with conventional moments. A comparison of Tchebichef and Legendre moments can be found in [5].

This paper analyzes some of the computational aspects of Tchebichef moments with the aim of improving the quality of the reconstructed images. It also recommends the use of orthonormal moments as against scaled Tchebichef moments to minimize the occurrences of numerical instabilities when the moment order becomes large. The paper also proposes moment renormalization for increased stability and accuracy of the results. Experimental results showing the comparison of the reconstruction errors using Legendre, Zernike, Scaled Techebichef and Orthonormal Moments are also given.

2. Scaled Tchebichef Moments

The Tchebichef moments of order $p+q$ of an image $f(i, j)$ are defined based on the scaled orthogonal Tchebichef polynomials $t_n(i)$, as

$$T_{mn} = \frac{1}{\rho(m,N)\rho(n,N)} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} t_m(i)t_n(j)f(i,j)$$

$$m, n, = 0, 1, \dots N-1, \tag{1}$$

and has an exact image reconstruction formula (inverse moment transform),

$$f(i,j) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} T_{mn}t_m(i)t_n(j). \tag{2}$$

In equation (1), the polynomials $t_n(i)$, satisfy the recurrence formula:

$$(n+1)t_{n+1}(x) - (2n+1)(2x-N+1)t_n(x)$$
$$+ n(1-n^2/N^2)t_{n-1}(x) = 0$$
$$n = 1,2 \dots N-2; \quad x = 0,1,\dots N-1. \tag{3}$$

with the initial conditions

$$t_0(x) = 1,$$
$$t_1(x) = (2x-N+1)/N. \tag{4}$$

and the squared-norm $\rho(n, N)$ is given by

$$\rho(n,N) = \frac{N\left(1-\frac{1}{N^2}\right)\left(1-\frac{2^2}{N^2}\right)\cdots\left(1-\frac{n^2}{N^2}\right)}{2n+1}$$

,
$$n = 0, 1, \dots, N-1. \tag{5}$$

For a detailed description of the Tchebichef moment equations, refer [1]. The inverse moment transform given by

$$f(i,j) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} T_{mn}t_m(i)t_n(j). \tag{6}$$

allows us to reconstruct the image intensity distribution from the computed moments.

3. Numerical Instability

In many practical applications, equation (6) is often approximated by the following equation:

$$\tilde{f}(i,j) = \sum_{m=0}^{M}\sum_{n=0}^{M} T_{mn}t_m(i)t_n(j). \tag{7}$$

where M denotes the maximum degree of polynomials used in the reconstruction of the intensity values. The value of M is usually considerably less than N–1. For example, Fig.1(a) shows a gray level image of size 200x200 pixels (N=200), and Fig.1(b) shows the image reconstructed image using moments up to a maximum order of 100 (i.e., M=100).

Fig. 1(a). Original image of size 200x200 pixels

Fig. 1(b). Reconstructed image with M=100.

The reconstruction error between the original image $f(i, j)$ and the reconstructed image $\tilde{f}(i,j)$ can be defined as

$$\varepsilon = \sqrt{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left\{f(i,j)-\tilde{f}(i,j)\right\}^2}, \tag{8}$$

and the plot of the reconstruction error for the image in Fig. 1(b) is given below.

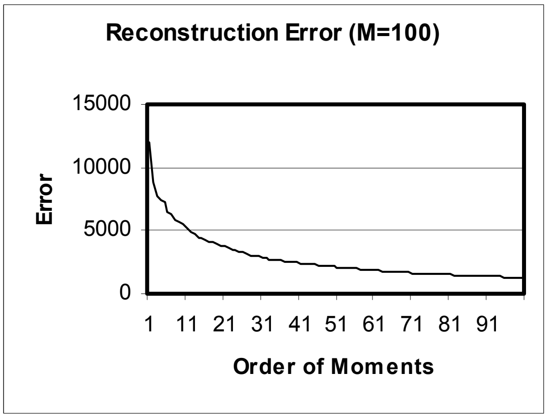

Fig. 2. Plot of reconstruction error.

The graph in the above figure is expected to tend to zero as M approaches the value of N–1.

Even though the scale factor introduced in [1] takes care of the numerical instabilities in the computation of low order Tchebichef polynomials, the recurrence relation given in (3) was found to propagate any numerical inaccuracies to polynomials of higher degree. Most importantly, the value of the squared norm in (5) tends to zero as n increases. Fig. 3 shows the plot of $\rho(n, N)$ with respect to n for N=200.

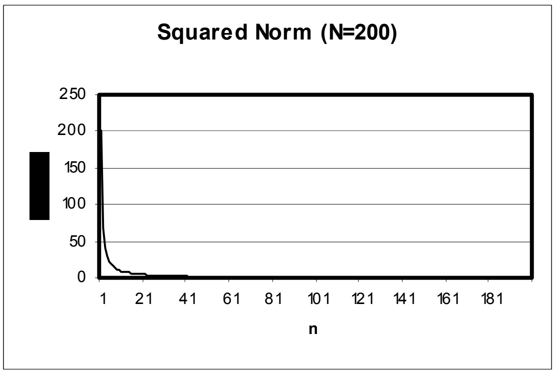

Fig. 3. Plot of $\rho(n, N)$.

As a result, the moments computed according to (1) assumes very large values when either m or n is large. The numerical instability in the computed moments is immediately reflected in the reconstruction error (Fig. 4) as a sharp increase in the error magnitude.

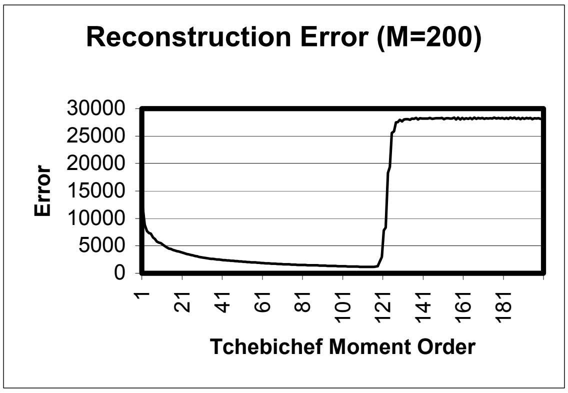

Fig. 4. Reconstruction error for large M.

This paper proposes methods to avoid the aforesaid numerical instabilities, so that very accurate results can be obtained while computing moments of large orders and reconstructing images from moments.

4. Orthonormal Moments

The problems associated with magnitude variations of the squared norm can be resolved by using orthonormal versions of the Tchebichef moments. They are defined using the following recurrence relation:

$$\hat{t}_n(x) = \alpha(2x+1-N)\hat{t}_{n-1}(x) + \beta\,\hat{t}_{n-2}(x),$$
$$n = 0, 1, \ldots N-2; \qquad x = 0,1,\ldots N-1. \qquad (9)$$

where

$$\alpha = \frac{\sqrt{4n^2-1}}{n\sqrt{N^2-n^2}},$$

$$\beta = -\frac{(n-1)\sqrt{2n+1}\sqrt{N^2-(n-1)^2}}{n\sqrt{2n-3}\sqrt{N^2-n^2}}$$

The initial conditions of the above recurrence relation are

$$\hat{t}_0(x) = N^{-1/2}$$

$$\hat{t}_1(x) = \frac{\sqrt{3}\;(2x+1-N)}{\sqrt{N(N^2-1)}}$$

(10)

The discrete orthonormal polynomials defined as above satisfy the following condition for all n:

$$\rho(n,N) = \sum_{i=0}^{N-1} \left\{\hat{t}_n(i)\right\}^2 = 1.0 \tag{11}$$

Experimental results have shown that orthonormalization has led to only a marginal improvement in the accuracy of the reconstructed images for large M (Fig. 5). This is because the recurrence relation (9) continues to propagate numerical errors, eventually leading to the instability observed in the previous case.

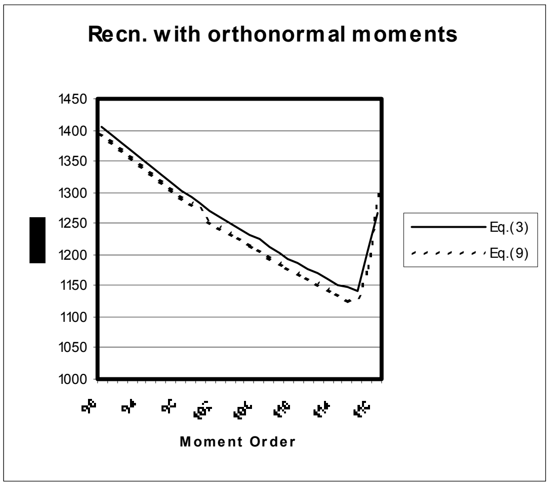

Fig. 5. Comparison of reconstruction errors showing a marginal improvement with orthonormal moments.

Since the new set of polynomials given in (9) has unit squared norm, we can re-normalise them to counteract any propagation of numerical errors through the recurrence relation, as follows:

$$\hat{t}_n(x) \leftarrow \frac{\hat{t}_n(x)}{\sqrt{\sum_{x=0}^{N-1} \left\{\hat{t}_n(x)\right\}^2}} \tag{12}$$

The renormalization of the computed moments is one step towards significantly improving the reconstruction accuracy. The following figure gives a comparison of the reconstructed errors with the new approach as against the plot given in Fig.4.

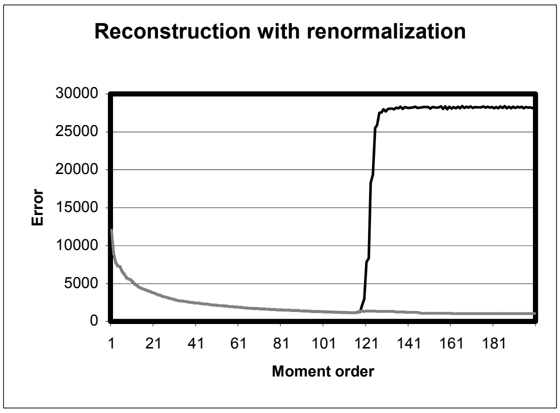

Fig. 6. The significant improvement with renormalization can be seen in the dotted-line graph.

Conclusions

The power of discrete orthogonal moments lies in their capability to represent image shape features, without the need for using approximation techniques as in the case of Legendre and Zernike moments. Further, the input image can the theoretically reconstructed from a sufficiently large number of computed moments. However, the recurrence relations used in the computation of Tchebichef moments cause the propagation of small numerical errors to higher order moments, severely affecting the accuracy of the reconstructed images.

This paper has introduced the theoretical framework for the computation of orthonormal Tchebichef moments, and used renormalization of moments to limit the propagation of numerical errors in the computation of higher order moments. Orthonormalization of Tchebichef moments has yielded some improvement in the accuracy of the computed moments, and in the quality of the reconstructed image.

Future work in this area is directed towards the minimization of the use of the recurrence relation by exploiting additional properties of Tchebichef moments. Evaluating boundary conditions directly, and using multi-step recurrence can possibly improve the results further. The symmetry property of Tchebichef moments can be combined with the recurrence relations in both x and y to reduce the area of the domain where the moment terms are computed.

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